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A STUDY OF THE RELATIONSHIP OF CENTRAL PRESSURE TENDENCY TO VARIOUS SYNOPTIC ASPECTS OF A CYCLONE

by

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by
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PREFACE

The purpose of this research was to examine quantitatively the possible effect of the slope of the core of a cyclonic system on the pressure tendency at the center and the further relationship, to that same tendency, of various other synoptic values available to the forecaster.

This work was conducted at the U. S. Naval Postgraduate School, Monterey, California, during the period November 1948 to May 1949.

I wish to express my appreciation to Professor William D. Duthie and Associate Professor A. B. Mewborn for their advice and guidance throughout.

Kenneth E. Wilson

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TABLE OF SYMBOLS

Δp - Pressure change, during a twenty-four hour period following map time, at the center of the system.

D_1 - Horizontal distance between the centers of a cyclone at the surface and 700mb levels.

D_2 - Horizontal distance between the centers of a cyclone at the surface and 850mb levels.

P - Observed central pressure of the system.

V - Speed of the center of the system.

V_x, V_y - E-W and N-S components of V

p_p - Pressure at any level
p_o - Pressure at surface
p_{15} - Pressure at 15,000 ft.
β - Dry adiabatic lapse rate
α - Actual lapse rate
e - A term involving and representing non-adiabatic temperature changes
T - Temperature
T_{na} - Temperature resulting from non-adiabatic processes

x, y, z, u, v, ω, ρ, e, g, t - Standard meteorological definitions

r, σ, S - Accepted statistical definitions

INTRODUCTION

The variation of pressure with time in any meteorological situation has long been recognized as one of the basic foundations upon which prognosis and forecast must rely. As a result, a rather extensive amount of meteorological literature has been devoted to the tendency equation in its various forms, and possible kinematic applications thereof. From a qualitative standpoint, the theory has been well developed. Quantitative studies have, however, progressed more slowly under a number of widely recognized handicaps. Although the principal work has been done on the local pressure variation, e.g. the recent work of Godson (6), certain of the rules developed, such as that of Petterssen (11), are concerned with the deepening or filling of the center of the system itself. These results require either moderate to extended computations, data not ordinarily available under routine conditions, or are applicable, with a sufficient degree of accuracy, for only relatively short range forecasts. The practicing meteorologist, under pressure of a fixed forecast deadline and with a forecast period of at least twenty-four hours, finds that he must still rely on experience and theoretical judgement in determining the probable pressure variation to be expected of a given system.

This study was undertaken to ascertain whether some method of quantitative assistance could not be uncovered through statistical inquiry. In order that any favorable results might be of use to the practicing forecaster, the following conditions were established:

(a) Availability of Data - That such data as might be necessary for the obtaining of the desired results should be included in that presented by the several maps customarily analyzed in the average meteorological office. Further, that it should not be limited by being among that data received from only selected stations, i.e., radiosonde or pibal reports.

(b) Ease of Application - That the results be adaptable to representation in such a form that extended computations would not be necessary.

(c) Extended Period of Application - That any results be valid for a period of at least twenty-four hours since that is the usual prognostic interval.

With these limitations in mind, and in view of the fact that the future intensity of a particular cyclone is ordinarily one of the major items of interest to the "customer" of the meteorologist, the investigation was confined to the future deepening or filling of the centers of low pressure systems.

For certain theoretical reasons, the original study was directed towards a determination of the possible correlation between such deepening or filling and the slope of the core of the cyclonic center as expressed by the horizontal distance between the surface center and the corresponding centers at the 700mb and 850mb levels. The assumption was made that this core was a straight line, at least from the surface to the 700mb level. Although such an assumption was believed to be incorrect, it was felt that the error introduced by variation from that condition might prove to be sufficiently small as to be negligible. Such was shown to be the case. The investigation was later expanded so as to inquire into the possible individual or further correlation of the following additional factors:

(a) The pressure at the center of the system.

(b) The speed of the system.

(c) The E-W component of the speed.

(d) The N-S component of the speed.

(e) The horizontal distance between the actual position of the center at the 850mb level and the position indicated by the assumption of a straight core from the surface to 700mb.

(f) The deepening or filling of the system during the previous twenty-four hours.

It soon became apparent to the author that a certain mutual correlation existed between the prospective deepening or filling of the system, the originally established criterion, and the existing pressure of the cyclone's center. At this time (b) - (f), above, were re-examined to determine their possible connection with the discrepancies between the calculated pressure change, during the ensuing twenty-four hour period, and that observed to occur.

As it developed, the investigation, based originally on the work of Bjerknes and Holmboe (2), divided itself into four parts as follows:

(1) To ascertain whether there existed any particular distance and hence any critical slope angle of the core, which would serve as a line of demarcation between future negative and positive pressure changes.

(2) To determine whether there existed any easily expressed relationship between the slope, expressed as a distance, and the magnitude of the pressure change.

(3) If (2) was affirmatively concluded, whether any of the other

variables would satisfactorily explain the remaining discrepancies.

(4) And finally, if (3) yielded positive results, whether any third variable could account for the residual error.

Since this was a statistical investigation, the resulting relationships are presented in terms of simple and multiple correlation coefficients, two, three, and four-variate regression equations and standard errors of estimate.

Data was assembled from maps covering a period of six consecutive winter months, October to March inclusive. Map time for the surface analysis was 0030Z, for the upper levels 0330Z, and for the limits of the twenty-four hour period 0030Z; the reasons for such choice of times will be discussed later. Sufficient cases were included so as to render the results, if not conclusive, at least reasonably reliable on the basis of standard sampling theory. In view of the intention to limit all computations and data to within that reasonably to be expected of a practicing aerologist, a maximum of three-figure accuracy was used -- two-figure where the probable accuracy of analysis and visual subdivision of measurements would so indicate.

With the above considerations in view, the following results of the investigation are advanced:

(1) That, within rather small limits of error, there does exist a critical value of the horizontal distance between the centers of the cyclone on the surface and at the 700mb level with respect to the future difference of pressure at the center of the system. This value for D_1 is 2.83°Lat. or 170 miles. For D_1 less than that value, the system may be expected to have filled at the end of the

next twenty-four hours for D_1 greater than 170 miles to have deepened.

(2) That the correlation coefficient between D_1 and Δp is large and negative, being <u>-.70</u> and that the multiple correlation coefficient of Δp based on D_1 and the original pressure at the center of the system is large and positive, <u>.93</u>. These correlations, especially the latter, indicate a pronounced relationship among the three variates and a reasonable degree of reliability might be expected of the regression coefficients computed therefrom. The corresponding standard errors of estimate are 6.34 mb and 5.2 mb and these are rather large in view of the mean value of Δp, which is -4 mb. However, since Δp varied from a +12mb to -25 mb, a range of 37 mb, the latter standard error of estimate defines a limit of 25-30% of the range. It might be pointed out that a more proper comparison may be made by the use of actual pressure rather than Δp. In this case, since the extremes of surface pressure may be considered to be 850 mb and 1050 mb, a dispersion of 200 mb, either S_p represents an error of only about 3.0%.

(3) That, through the use of the computed regression equations or the corresponding graph, the analyst may reasonably expect to forecast the central pressure of a cyclone twenty-four hours in advance with the probability being .50 that his predicted pressure will be within 5 mb of that observed at the end of the period. It is believed that, within this not unreasonable limitation, such a result is of some assistance in quantitative forecasting.

CHAPTER I

EARLIER INVESTIGATIONS OF PRESSURE TENDENCY

1. Theoretical derivations.

The first, and what still might be called the classical, development of the tendency equation was carried out by Margules (9) in 1904. The later treatment by Bjerknes (1) and Holmboe (7) differ only in notation. Beginning with the hydrostatic equation and advancing through familiar steps, we arrive at the equally familiar:

$$\frac{\partial p_z}{\partial t} = -\int_z^{\infty} g\left(u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y}\right) dz \tag{1}$$
$$-\int_z^{\infty} g\rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) dz + g\rho_z\,\omega_z$$

which, for the surface pressure tendency, reduces to

$$\frac{\partial p_0}{\partial t} = -\int_0^{\infty} g\left(u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y}\right) dz \tag{2}$$
$$-\int_0^{\infty} g\rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) dz$$

on the recognition of the fact that there can be no vertical motion through the level of the surface of the earth.

On the same basis, but through a different evaluation of the hydrostatic equation, Houghton and Austin arrived at the following expression for the surface pressure tendency:

$$\frac{\partial p_0}{\partial t} = -\int_0^{\infty} g\left(u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y}\right) dz \tag{3}$$
$$+\int_0^{\infty} g\omega\left(\frac{d\rho}{dz} - \frac{\partial \rho}{\partial z}\right) dz$$

in which, they have shown, the two terms on the right side of the equation are of the same order of magnitude.

An adaption by Raethjen (12) of an original derivation by Panofsky (10) provides us with a third equation. This development is based on the series of identities:

(4)
$$\frac{\partial}{\partial z}\frac{\partial}{\partial t}\ln p = \frac{\partial}{\partial t}\frac{\partial}{\partial z}\ln p = \frac{\partial}{\partial t}\left(-\frac{g}{RT}\right)$$
$$= \frac{g}{RT^2}\frac{\partial T}{\partial t}$$

and results in the following expression for the surface pressure tendency:

(5)
$$\frac{\partial p_0}{\partial t} = \frac{g p_0}{R}\int_0^{\infty}\frac{1}{T^2}\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)dz$$
$$+ \frac{g p_0}{R}\int_0^{\infty}\frac{\omega(\Gamma - \alpha) - e}{T^2}\,dz$$

where Γ is the dry adiabatic lapse rate, α the actual lapse rate, and the e corresponds to an expression involving $\frac{dT_{ne}}{dt}$ and represents the results of non-adiabatic temperature changes.

Recent work by Godson (6) has produced still another result which he chooses to call the "isobaric tendency equation". Here, from the mathematical sequence of :

(6)
$$\frac{\partial}{\partial z}\left(\frac{1}{\rho}\frac{\partial p}{\partial t}\right) = \frac{\partial}{\partial z}\left(RT\frac{\partial \ln p}{\partial t}\right)$$
$$= \frac{g}{T}\frac{\partial T}{\partial t} + \frac{R}{P}\frac{\partial T}{\partial z}\frac{\partial p}{\partial t}$$

he arrives at an expression for the surface pressure tendency as follows:

$$\frac{\partial p_0}{\partial t} = g\rho_0 \int_0^\infty \left(u \frac{\partial}{\partial x} \ln T + v \frac{\partial}{\partial y} \ln T \right) dz$$

$$(7) \qquad - g\rho_0 \int_0^\infty \left(\frac{d}{dp} \ln T - \frac{\partial}{\partial p} \ln T \right) \frac{dp}{dt}\, dz$$

$$- g\rho_0 \int_0^\infty \frac{d}{dt} \ln T_{na}\, dz$$

2. Quantitative approaches.

There have been at least three proposals for the reduction of pressure tendency to a calculable status:

(a) The technique of Rossby as applied to the "classical" equation after the inclusion of certain limiting assumptions by Bjerknes and Holmboe (2).

(b) Godson's (6) evaluations of his "isobaric" tendency equation.

(c) Certain rules developed by Petterssen (11) from the general mathematical expression for the difference between the "local" and "individual" changes of any variable; in this case, pressure.

All of these lend themselves to practical application, to a varying degree. However, each fails, in at least one respect, to meet the requirements, as established for the purpose of this study, of a practical forecasting tool. A brief discussion of the details of each technique follows - and from this it will be seen that Petterssen's rules come the closest to fulfilling the desirable qualifications referred to in the Introduction.

The technique referred to in (a) is not strictly applicable to the problem under discussion, since it is concerned with the determination of the pressure tendency at some upper level, based on the tendency at the ground. However, it does serve as an example of early efforts to correct the tendency equation into a form susceptible to mechanical computations. In reference to the "classical" tendency equation:

$$\left(\frac{\partial p}{\partial t}\right)_0 - \left(\frac{\partial p}{\partial t}\right)_\phi = -\int_0^\phi V \cdot \nabla_H \rho \, d\phi - \int_0^\phi \rho \nabla_H \cdot V \, d\phi - (g \rho V_z)_\phi \tag{8}$$

Bjerknes and Holmboe (2) attempted to show that, with certain assumptions, the last two terms on the right side were of opposite sign and that their joint contributions could be neglected reducing (8) to the one or "advective" term. Rossby then derived from this an equation for quantitative solution of pressure tendency. In so doing he introduced further assumptions, among them that of geostrophic wind. Unfortunately, both of these developments were later shown to be invalid. However, even if valid, Rossby's equation would not have been acceptable under the conditions established for this investigation since it involved the area under a wind-shear hodograph from 3,000 - 10,000 feet. This restricted its application to stations where such upper air soundings were available.

Godson (6) gives an example of the computation of the surface tendency based on the observed tendency at 15,000 feet. From equation (7), he develops the applicable form to be:

$$\frac{\partial p_0}{\partial t} = \frac{\rho_0}{\rho_{15}} \frac{\partial p_{15}}{\partial t} + g\rho_0 \int_0^{15} \left(u \frac{\delta \ln T}{\delta x} + v \frac{\delta \ln T}{\delta y} \right) dz$$

$$(10) \qquad - g\rho_0 \int_0^{15} \left(\frac{d \ln T}{dp} - \frac{\delta \ln T}{\delta p} \right) \frac{dp}{dt} \, dz$$

$$- g\rho_0 \int_0^{15} \frac{d}{dt} \ln T_{na} \, dz$$

In evaluating the several terms, he finds it necessary to use

a) A graph of values of P_{15}, observed at two-hour intervals,

b) The wind hodograph from the friction layer to 15,000 feet corrected for non-geostraphic wind values,

c) The appropriate tephigram, and

d) Elsasser's radiation chart

with supplementary computations in each case. The necessity for such data and calculations, in addition to its short period of reliability, disqualifies this technique under that same set of limitations advanced earlier.

We arrive, then, at Peterssen's (11) adaption of equation (2) to quantitative use by its utilization only at points where the convective term $(c \cdot \nabla p)$ vanishes. While it is undoubtedly true that, at such points, the instantaneous "local" and "individual" changes are equal, that is

$$(11) \qquad \frac{\delta p}{\delta t} = \frac{\partial p}{\partial t}$$

the extension of the time interval introduces the obvious fallacy of strict adherence to extrapolation.

It is recognized that no claim was made in either of the first two cases other than as supplementary computations leading to rough first-approximations in support of dynamic theory, nor on the part of the third to other than qualitative interpretation.

CHAPTER II

DETAILS OF THE INVESTIGATION

1. Organization of data.

The period covered by the investigation was from October, 1945 through March, 1946. The charts used were the 0030 surface (sea level) and the 0330 upper air (700 mb and 850 mb constant pressure). These charts were obtained from the chart file of the Department of Aerology, U. S. Naval Postgraduate School, Monterey, California. In order to compensate for the time difference between the two maps, the position of the surface center was advanced one-seventh of its observed travel during the following twenty-four hour period. This was done in the interest of investigative procedure although it is the author's belief, as will be discussed, that this was not necessary.

In selecting the cyclones to be used in the investigation the following precautions were observed:

a) The charts were carefully examined with the purpose of ensuring the correctness of the analysis.

b) The surface centers were enclosed by at least one isobar and were sufficiently well defined as to permit the location of the center with reasonable accuracy.

c) The centers on the upper air charts were, in the majority of cases, enclosed by one or more isohypses. However, in a few instances, where an abnormally wide spacing of isohypses and one or more reports permitted the reasonable location of a center not encircled by a standard isphypse, such a center was used.

d) In general, only those areas of the charts were used where adequate reports guaranteed the certainty of the analysis in both the surface and upper air charts. However, in order to test the extension of the theory to two areas of meteorological importance not meeting this qualification, a number of cases in the Aleutian Islands-Alaskan Gulf and Newfoundland-Nova Scotia regions were included.

A total of eighty-four surface cases were tested. Of these, seventy-five had a corresponding center on the 700 mb chart, seventy had a corresponding center on the 850 mb chart, and sixty-one had centers on both levels, associated with the particular surface cyclone. The following data, where applicable, were recorded or computed in each case, with all distances being expressed in degrees of latitude:

a) date-time group (for identification)

b) surface center - latitude

c) surface center - longitude

d) surface center - central pressure

e) surface center - V_x

(Since the distance was expected in the majority of cases to be to the west, this direction was taken as positive for all latitudinal velocities and distances -- despite its contradiction of the accepted custom).

f) surface center - V_y

g) surface center - corrected latitude

h) surface center - corrected longitude

i) 850 mb level center - latitude

j) 850 mb level center - longitude

k) 850 mb level center - V_x

l) 850 mb level center - V_y

m) 700 mb level center - latitude

n) 700 mb level center - longitude

o) 700 mb level center - V_x

p) 700 mb level center - V_y

q) 850 mb to surface - difference of latitude

r) 850 mb to surface - difference of longitude

s) 700 mb to surface - difference of latitude

t) 700 mb to surface - difference of longitude

u) horizontal distance - 850 mb to surface

v) horizontal distance - 700 mb to surface

w) pressure change - next twenty-four hours

From among these, or items computed from them, were obtained the variates to be used in the several correlations carried out.

2. Tetrachoric distributions.

On the basis of Bjerknes (1) original work on waves in the westerlies, a discussion of which may be found in any advanced text, it is reasonable to predict a probable marked degree of relationship between the horizontal distance of the surface center of a cyclone from the center at any upper level; i.e., the slope of the core of the system, and the magnitude and sign of the value for ∇p given by the standard tendency equation. Qualitatively, the reasoning may be expressed in this manner.

a) The divergence term depends on the distribution of divergence and convergence above the point under consideration, the line of demarcation being the level of the last closed isobar. This level, for a given system will have a close relation to the horizontal distance between the surface and upper centers. From the nature of the dependency, it might be expected that a decrease in the magnitude of the distance would result in an increase in the algebraic value of the pressure tendency.

b) To a great extent, the advective term will depend on the thermal asymmetry or symmetry of the cyclone. Basically, such asymmetry will be interconnected with the same distance as above. It might further be expected that the nature of the interaction would be the same as in (a).

On the basis of (a) and (b), the reasonable assumption was made that the total pressure tendency at its center would vary directly with the slope of the core of a cyclone, or inversely with the horizontal distance between the centers of any two layers, and that there might be some critical value of the latter at which the corresponding Δp would change from negative to positive. The layer over which the horizontal distance would be measured was chosen as the surface to the 700 mb level, since this was the thickest layer for which charts are usually drawn at each boundry and through which the center usually maintains its identity. The distance so measured was considered as D_I. The corresponding values of Δp and D_I were plotted, resulting in Fig. 1.

Since the initial interest was in the possibility of the existence of a critical value for D_I, this was determined, by inspection, to be 2.83° lat. (170 mi.) and produced the tetrachoric distribution shown:

	$D_1 < 2.83°$	$D_1 > 2.83°$
Δp (+)	15	6
Δp (−)	2	52

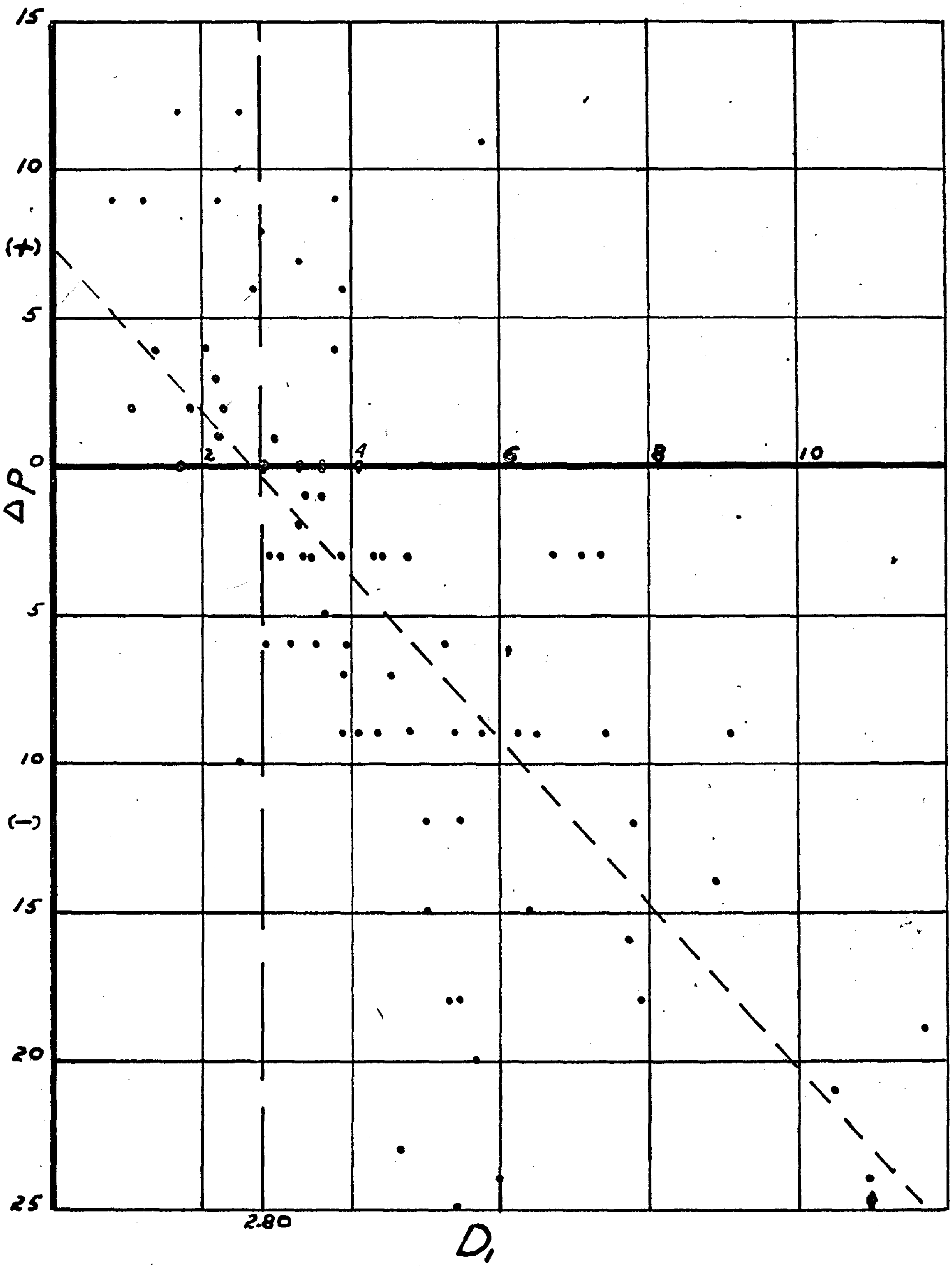


FIG 1

This distribution was evaluated in two ways. First, it might be considered that quadrents II and IV include the theoretically correct cases in that they support the initial assumption. On this basis, sixty-seven of the seventy-five cases, or 89.5%, successfully agree with the prognostic criterion. It might be pointed out that, if the best weighted estimate of quadrent distribution were used; i.e., in the proportions indicated by the end totals of columns and rows, the "correct" percentage would be 65.5%. If a pure chance or equal quadrant distribution were assumed, a percentage of 50.0% would result. Secondly, the improbability of such a distribution occurring by chance was expressed by the Skill Score Method* of Brier (3). The resulting Score was 73.8%. If the above weighted and chance distributions were substituted the Skill Scores would be 8.2% and 0.0% respectively, which even more markedly indicates the improvement of this distribution over either random or intelligent guesses.

Fig. 2 is a plot made of corresponding values of Δp and D_2, where D_2 represented the horizontal distance between the surface and 850 mb level centers. Here no critical distance of demarcation was apparent and no tetachoric distribution was possible. Possible explanations for this are that the layer is too thin to be indicative of the desired slope or that this level might still experience undue influence by surface friction and/or friction layer turbulence.

3. Two-variate correlations.

Since the relationship between D_1 and Δp presented at least a tetrachoric distribution, it was reasonable to inquire into the possibility of

*See Appendix for a description of the method of computation.

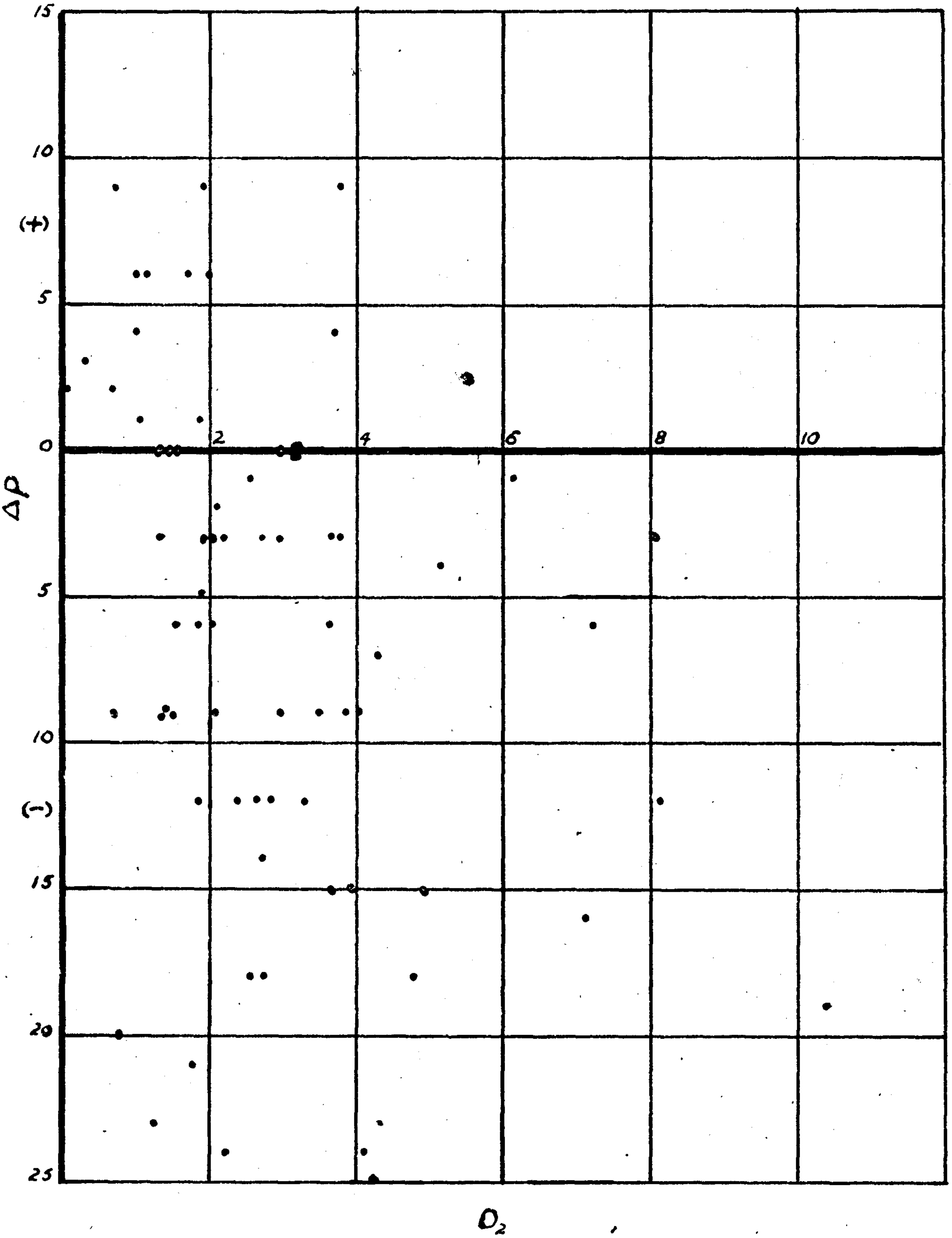


FIG 2.

a quantitative statistical correlation between the two variates. The scatter diagram resulting from the plotting of two variables in conjunction, Fig. 1, indicated a favorable prospect for such a possibility. Actual calculations produced the values for the following simple statistical functions:

(a) simple correlation coefficient, r, of the deepening or filling to the displacement between the two centers

(b) the arithmetic means of Δp and D_1

(c) the unit of standard deviation (variance from the mean), σ, and

(d) standard error of estimate, S.

$$(X_1 = \Delta p;\ X_2 = D_1)$$

$\bar{X}_1 = -4.83$ (mb)	$\bar{X}_2 = 4.43$ (°lat.)
$\sigma_1 = 9.19$	$\sigma_2 = 2.33$
$r_{12} = -.70$	$S_{12} = 6.52$ *

The regression equation of X_1 on X_2 as calculated from this data was:

$$X_1 = 2.76\ X_2 + 7.44 \tag{12}$$

and this regression line is plotted in Fig. 1. As a matter of investigative curiosity and, principally, as a preliminary to later multiple variate correlations, a number of further relationships were computed; namely,

X_1 - Δp - on

X_3 - D_2 - the distance between the system centers at surface and 850 mb level

X_5 - P - existing central surface pressure

X_8 - V - velocity of storm along its course for succeeding twenty-four hour period

* Details of computations may be found in Appendix

$X_2 - D_1$ on

$X_5 - P$

$X_8 - V$

$X_8 - V -$ on

$X_5 - P$

The simple statistical functions, as defined above, are listed below:

a) $\bar{X}_1 = -6.26$ (mb) $\bar{X}_3 = 2.86$ (°lat.)

$\sigma_1 = 8.59$ $\sigma_3 = 2.14$

$r_{13} = -.29$ $S_{13} = 8.25$

b) $\bar{X}_1 = -5.19$ (mb) $\bar{X}_5 = 993.82$ (mb)

$\sigma_1 = 9.04$ $\sigma_5 = 11.37$

$r_{15} = -.35$ $S_{15} = 8.50$

c) $\bar{X}_1 = -4.70$ (mb) $\bar{X}_8 = 9.43$ (°lat./24 hr)

$\sigma_1 = 9.34$ $\sigma_8 = 4.25$

$r_{18} = .33$ $S_{18} = 8.78$

d) $\bar{X}_2 = 4.43$ (°lat.) $\bar{X}_5 = 993.63$ (mb)

$\sigma_2 = 2.34$ $\sigma_5 = 11.51$

$r_{25} = .33$ $S_{25} = 2.20$

e) $\bar{X}_2 = 4.25$ (°lat.) $\bar{X}_8 = 9.53$ (°lat./24 hr)

$\sigma_2 = 2.32$ $\sigma_8 = 4.13$

$r_{28} = -.35$ $S_{28} = 2.18$

f) $\bar{X}_8 = 9.43$ (°lat/24 hr) $\bar{X}_5 = 990.25$ (mb)

$\sigma_8 = 4.25$ $\sigma_5 = 10.52$

$r_{85} = -.12$ $S_{85} = 4.25$

The scatter diagrams for the above pairs (except (a)) of inter-related variates are shown on the succeeding pages as Figs. 3 - 7.

In addition to these computations, a visual inspection was made of additional data to ascertain the possibility of further correlations. The following relationships were inspected:

X_1 - Δp - with

$X_4 - \left(\frac{D_1}{2} - D_2\right)$ - the horizontal distance between the actual center at the 850 mb level and the estimated center based on the assumption of a straight line core to the 700 mb level

X_6 - V_x - the y-component of the velocity of the system during the ensuing twenty-four hours

X_9 - $(\Delta p)'$ - the deepening or filling during the previous twenty-four hours

None of these showed evidence of sufficient correlation to warrant calculation of the statistical functions.

4. Multiple-variate correlations.

In order to complete the statistical investigations a number of multi-variate correlations were carried out. These, and their associated multiple correlation coefficient, were:

a) X_1 (Δp) on X_2 (D_1) and X_5 (P); $r_{1.25} = .93$ *

b) X_1 on X_2 and X_8 (V); $r_{1.28} = .71$

c) X_1 on X_5 and X_8; $r_{1.58} = .46$

d) X_1 on X_2, X_5 and X_8; $r_{1.285} = .77$

* Details of computations may be found in Appendix.

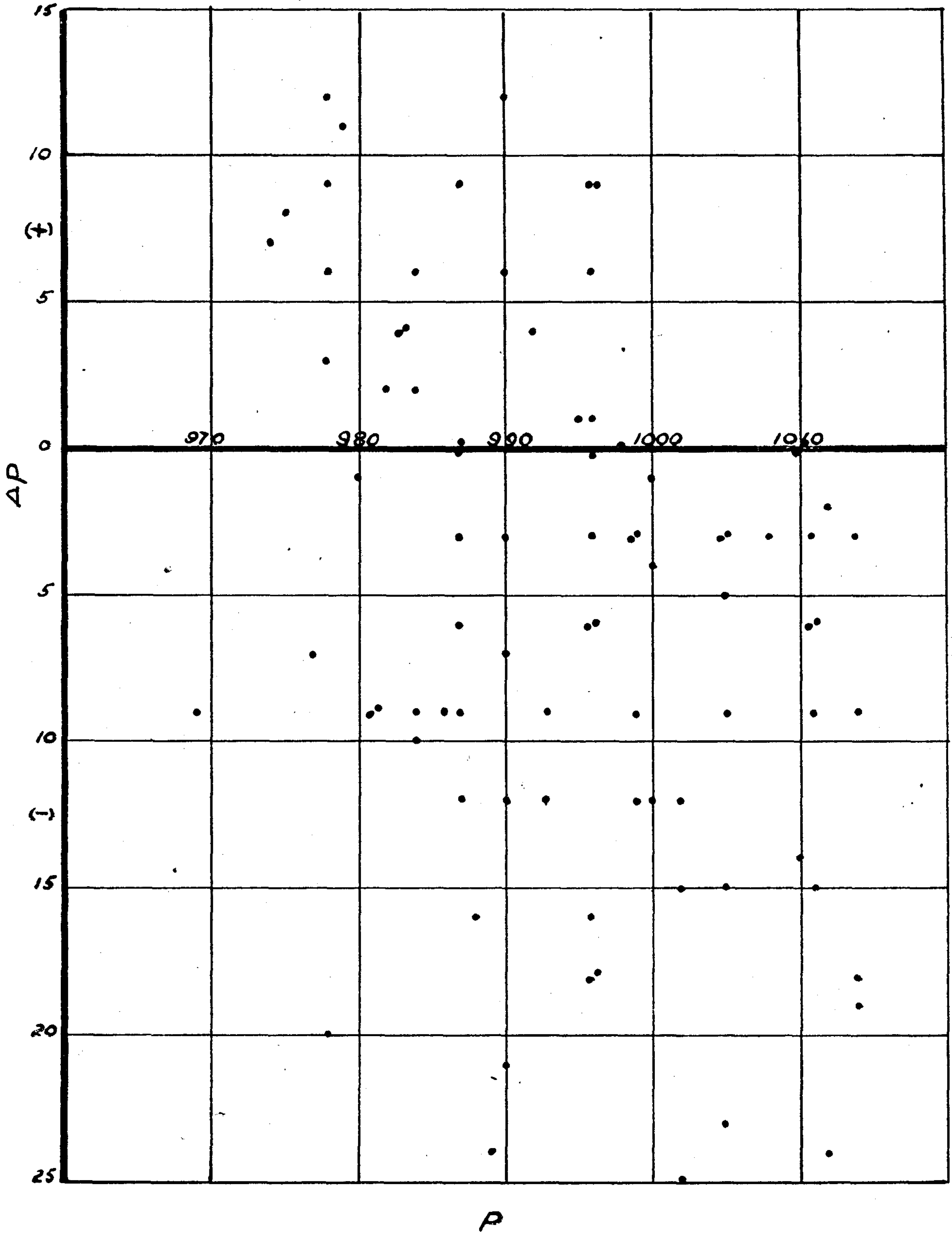
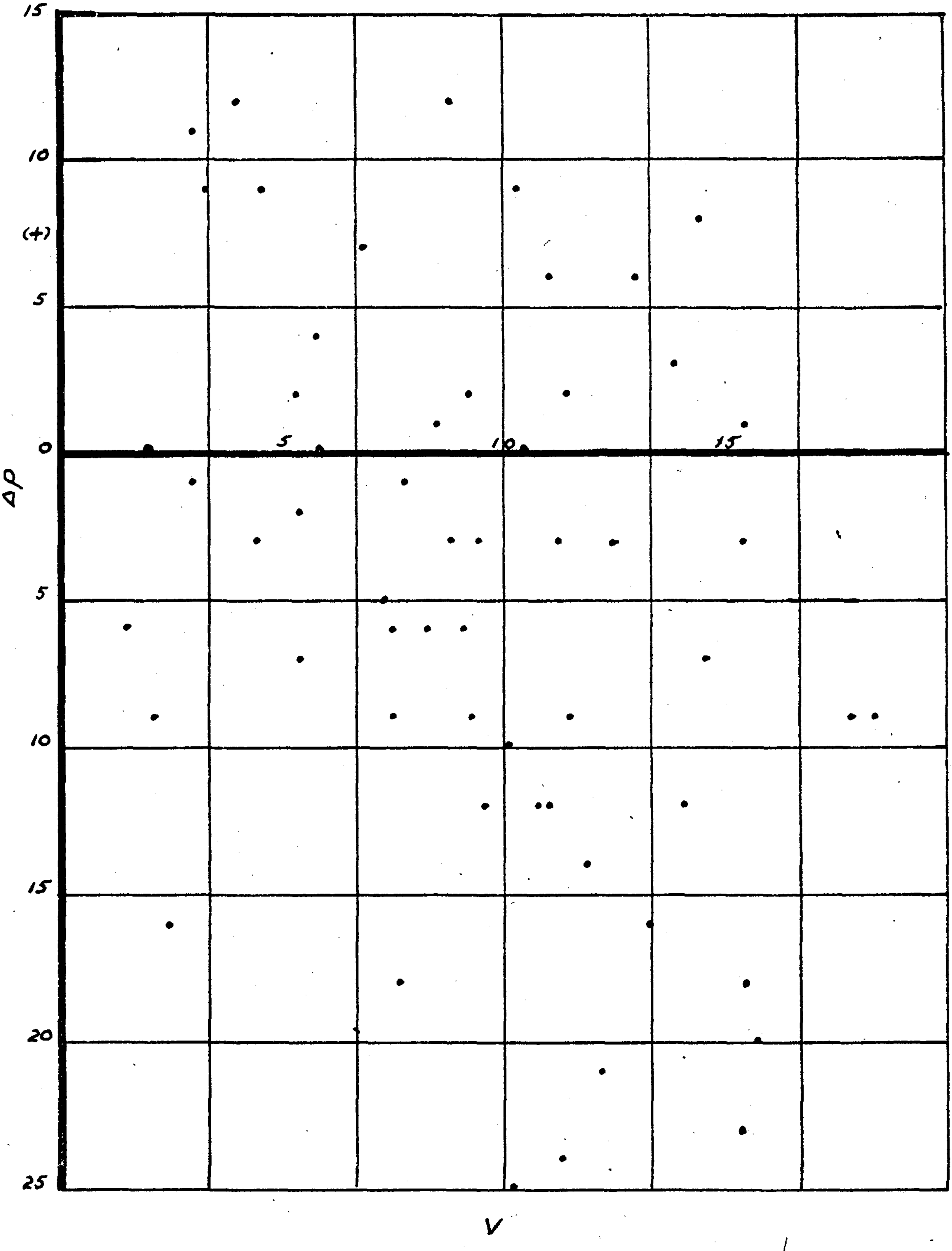


FIG 3



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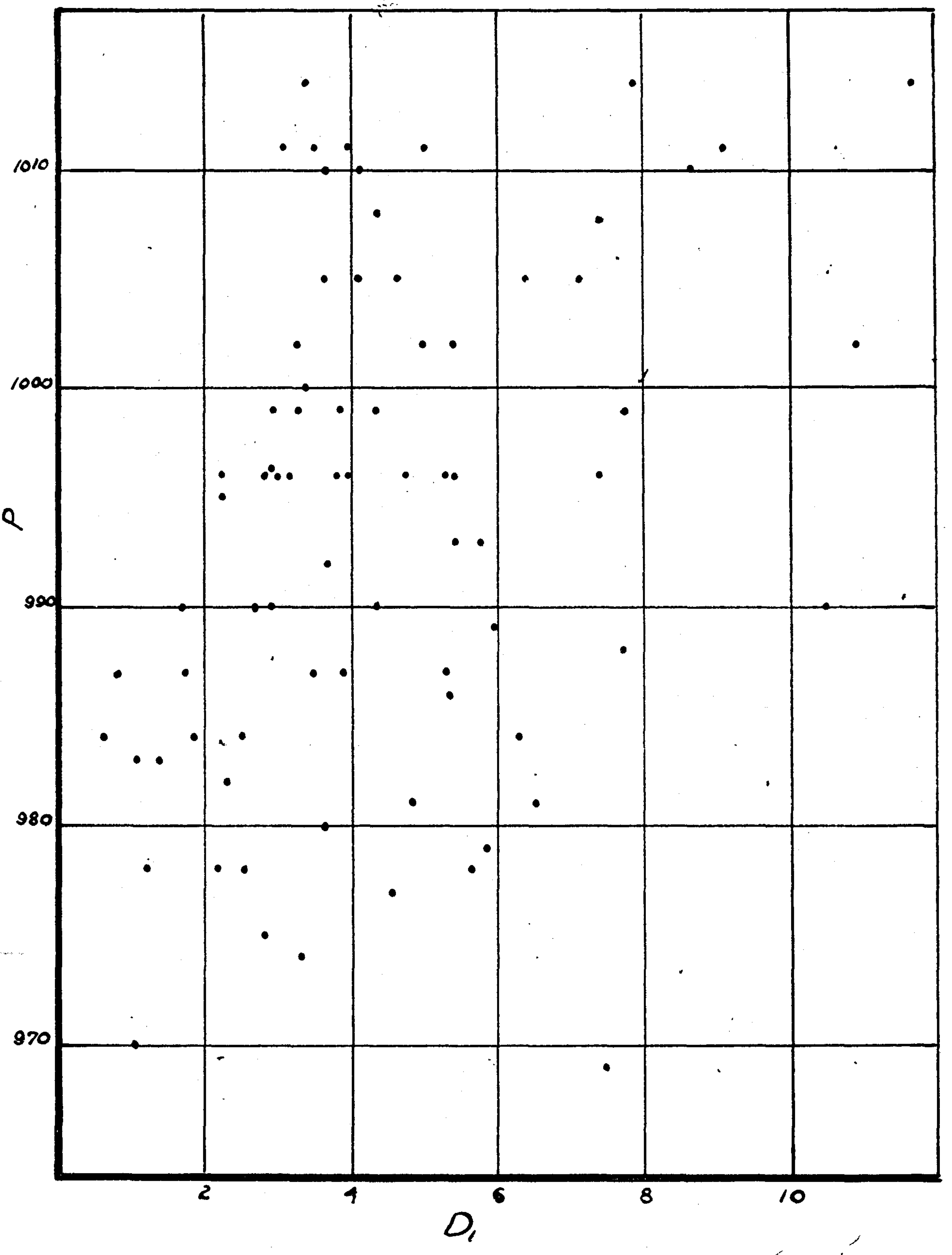
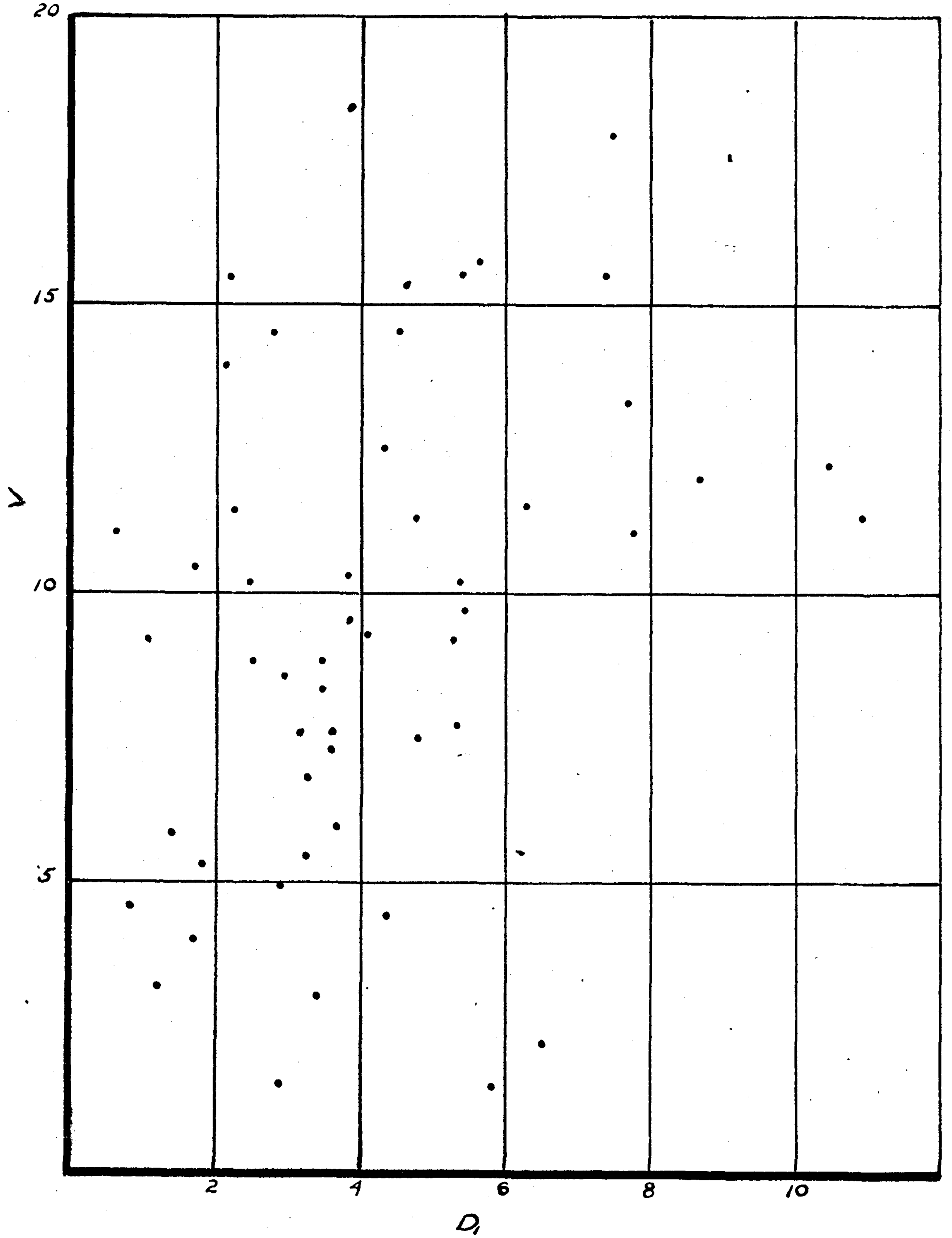


FIG 5



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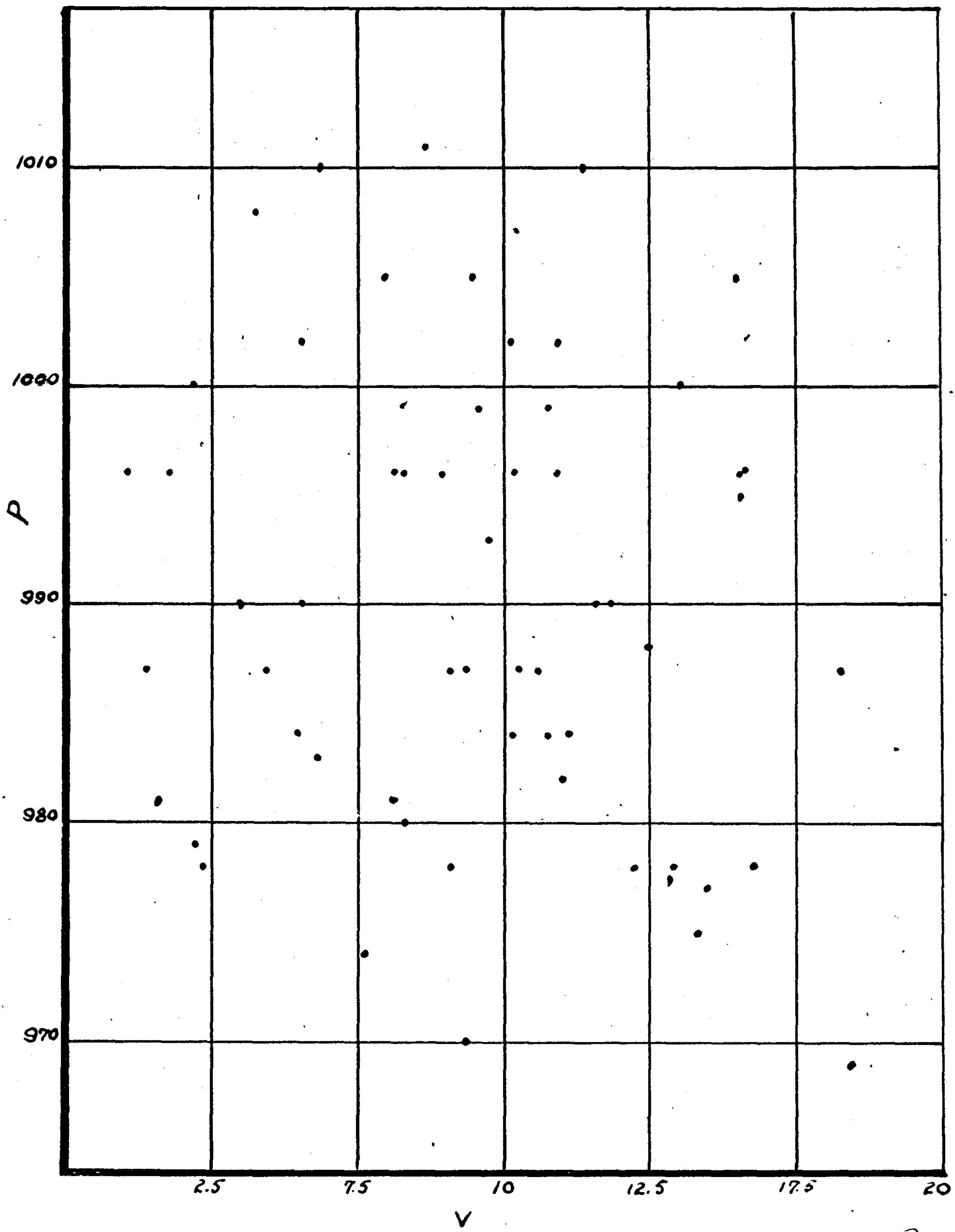

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The basis of these computations was to determine whether any combination of independent variates might interact in such a manner as to govern the expected values of the dependent variable more closely. With the exception of the original proposal of the existence of a relationship between the deepening or filling of the center of a system and the slope of its core, no attempt was made to predict, on the basis of theoretical considerations, the possible effects of the other variables acting either separately or jointly. This explains the rather random choice of variates with which the investigation was carried out - the choice being limited, in fact, only by the amount of information available to the forecaster on the two maps under consideration. In this connection, the question might be raised as to the propriety of using the velocity of the system during the twenty-four hours following the map time for the surface analysis, as both a variable and as the guide for the advancing of the surface system up to map time for the upper level analysis. Obviously, this fails to satisfy the original requirement that variates used as an argument in any forecasting technique developed be available to the analyst on the two maps utilized and not require prognostication themselves.

In accordance with statistical practice, a series of independent cases were tested to verify the results of the investigation. These will be discussed in full in the next chapter, it being sufficient to state at this point that in these test cases the velocity for the past twelve hours was extrapolated ahead for the three hours lapse between the surface and 700 mb maps. The difference in the majority of the cases in the value of Δp was found to be less than 2.5 mb from the value obtained by use of the future movement for advancing the surface system. In those instances where the discrepancy was larger than 2.5 mb; the number resulting in increased accuracy was slightly greater than those introducing added inaccuracy. The actual figures were as follows:

Number of random test cases	50
Number in which discrepancy $<$ 2.5 mb	31
Number in which discrepancy $>$ 2.5 mb but resulted in closer approach to observed (Δp)	11
Number in which discrepancy $>$ 2.5 mb and introduced less accurate result	8

CHAPTER III

SUMMARY OF CONCLUSIONS

1. Evaluation of results.

The principal conclusion that may be drawn from this study is the sustaining of the assumption that there is a high degree of correlation between the slope of the core of a cyclonic system and its future intensity trend -- and, further, that some critical value exists at which this trend changes from negative to positive. This latter may be substantiated from the evaluation of tetrachoric distribution of the previous chapter. A percentage accuracy of 89.5% and a Skill Score of 73.8% are sufficiently large to establish the credibility of the distribution and the existence of a critical distance of 2.83° latitude or 170 miles. With a distance of less than this amount, the system may, with a great deal of reliability, be predicted to fill and, conversely, with D_1 equal to or greater than that value, to deepen.

The computed two-variate correlations effectively served to establish more completely the inter-relation between the above two variables. The correlation coefficient was large and negative (-.70) and was markedly greater than the remaining coefficients, whose magnitudes changed from .12 to .35. While such a value of $r = -.70$ is customarily accepted as the lower limit of the statistically significant range, it represents in the field of meteorology a relatively important correlation, since the numerous intangible variables, acting simultaneously, reduce the threshold of credibility to a decidedly lower value. Because the correlation is negative, a decrease in the displacement will result in an increase in the future pressure. It may be seen from Fig. 1 and the magnitude of the units of standard deviation and the standard errors of estimate that considerable dispersion from a linear relation still remains. It was in an attempt to reduce this dispersion that the multi-variate correlations were undertaken.

With one exception, these multiple relationships did little to improve the situation, their magnitudes being .46, .71, and .77. Even the latter two values are not sufficient to warrant the added consideration of the second variable. However, the exception, which was the mutual consideration of the principal distance (from the above section), the existing pressure at the center and the intensity trend of the system, resulted in a multiple correlation coefficient of .93. This value is considerably better than any of the other relationships investigated and can be considered statistically significant without reservation. From this value of $r_{1.25}$, the following regression equation was computed:

$$X_1 = -3.62\,X_2 - .52\,(X_5 - 994) + 11.00 \tag{13}$$

where $X_1 = \Delta p$ during next twenty-four hours, in mb.

$X_2 = D_1$ displacement between surface and 700 mb. centers in degrees latitude.

and $X_5 = P$, existing central pressure of system, in mb.

This equation is readily expressed in graphical array, as in Fig. 8, for ease of utilization. By means of this graph, estimated values of X_1 were obtained for the original data and such estimated results were plotted against the observed pressure changes from that data, Fig. 9. It can be seen that the correlation is linear,

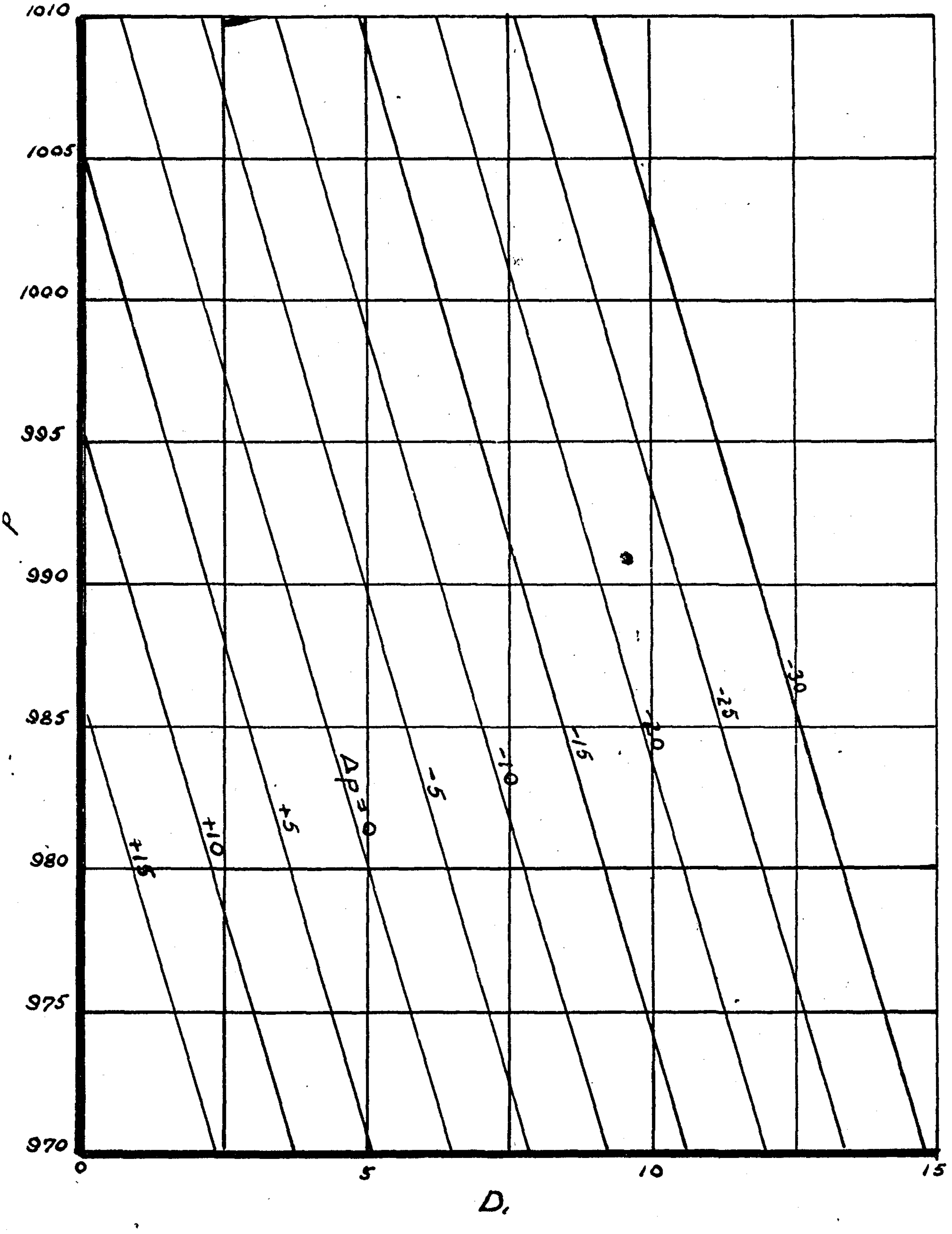
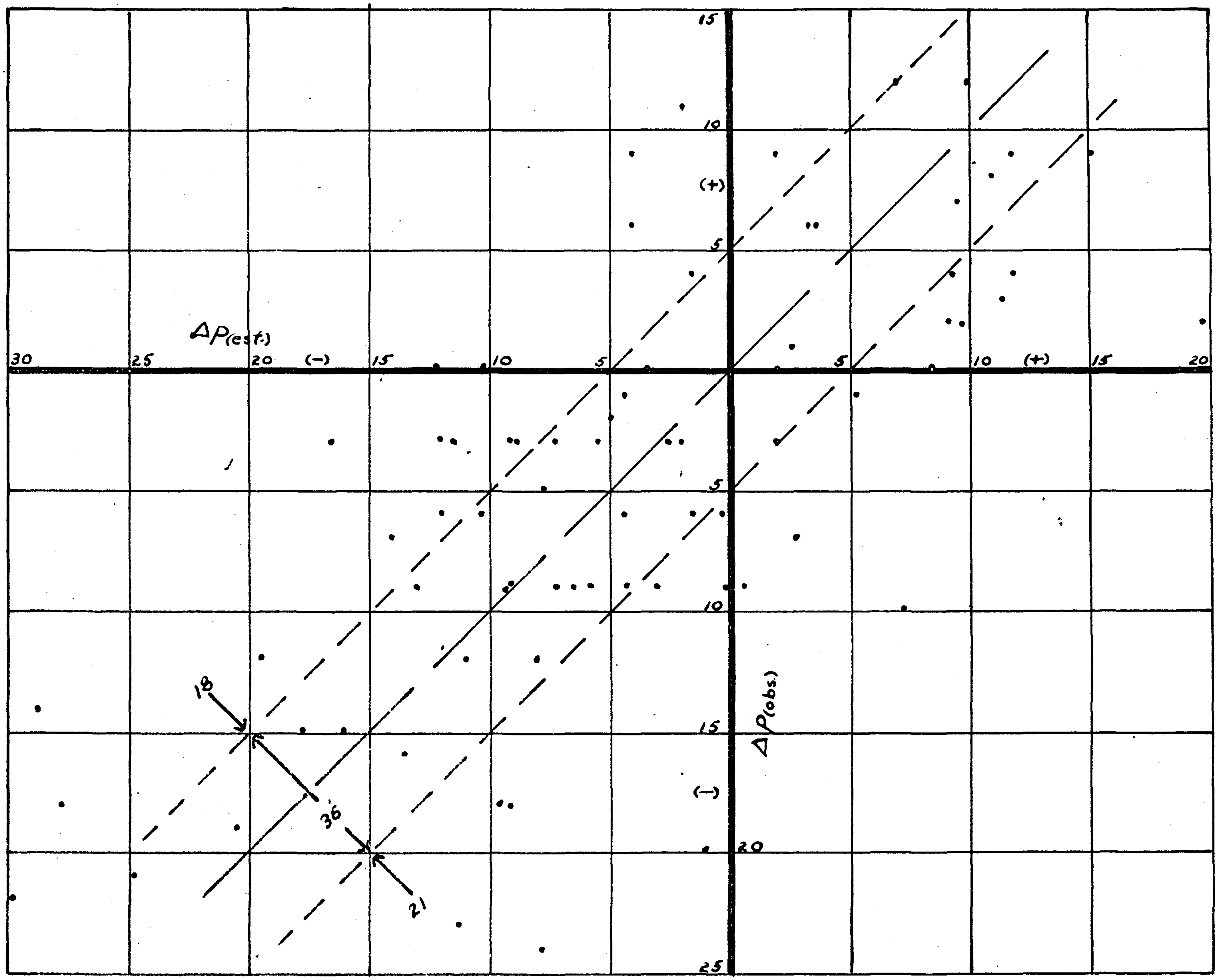

1010
1005
1000
995
990
985
980
975
970
P
0
5
10
15
D,
+15
+10
+5
ΔP = 0
-5
-10
-15
-20
-25
-30

FIG 8



$\Delta P_{(est.)}$
30
25
20
(–)
15
10
5
5
10
(+)
15
20
15
10
(+)
5
5
10
15
(–)
20
25
$\Delta P_{(obs.)}$
18
36
21

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the slope of the regression line equals 1.0 and that 50% of the cases lie within 5.2 mb. of that regression line. Admittedly, this is large compared with the mean pressure change, $\Delta p = -4$, mb., but (as was pointed out in the Introduction) compared to the range of either pressure change or the surface pressure (of the order of 200 mb) its magnitude is not so alarming. Certainly, the obtaining of an estimate of a central pressure, twenty-four hours in advance, which should be within 5 mb. half of the time is of some value, especially to the inexperienced forecaster. That this reliability is fairly consistent may be seen from the fifty supplementary cases. In Fig. 10, these are plotted with the same arguments as above. The regression line is the same and 50% of these test cases lie within 4.9 mb. This agrees closely with the original results and serves to emphasize the limits set by them.

2. The level of non-divergence.

According to the classical theory, the level of non-divergence coincides with the level of the last closed isobar over any given point. When the mass divergence above that level equals the mass convergence below, the pressure tendency at the surface will be zero. If, in equation (13), X_1, or Δp, is set equal to zero and certain assumptions are made concerning the intensity and dimension of a cyclone, then it is possible to calculate the corresponding height of the last closed isobar above the center. For this purpose, the following three cases were assumed:

Central pressure	-	1002 mb.	993 mb.	975 mb.
Diam. of surface system	-	300 mi.	600 mi.	1000 mi.
Diam of 700 mb. system	-	same as surface		

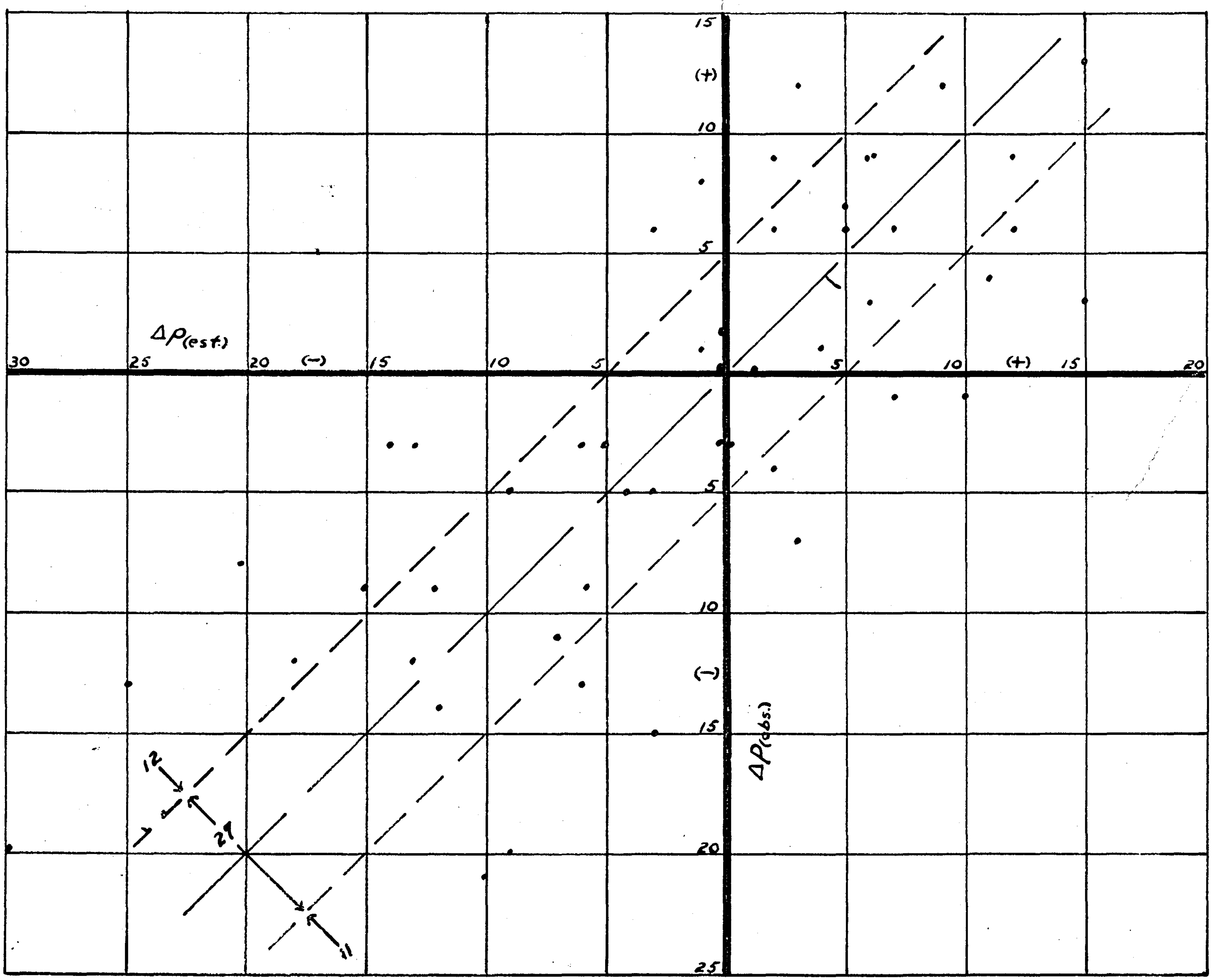

FIG 10.

Although the author has no statistics to support them, it is felt that these figures are quite representative of three stages of cyclonic development. The corresponding levels of non-divergence for these cases were computed to be 12,200 ft., 14,900 ft., and 13,700 ft., respectively. In this connection it might be pointed out that Charney (5) has shown that Rossby's formula for the speed of a barotropic wave will hold for baroclinic waves if the mean zonal wind is used and that this is about equivalent to the wind at 600 mb. He further quotes Holmboe to the effect that this is true only at the level of non-divergence, which also must be at approximately 600 mb. The data of both Willett (13) and V. Bjerknes (3) on the level of average zonal wind, and hence of non-divergence, agree quite closely on the value of 580 mb. or 4.25 km. (13,800 ft.). The close agreement of the computed values to the above serves in the nature of substantiation of the investigation. It is believed that the variation might have been introduced by the assumption that the diameters of the two centers are equal. It is the author's contention that many cases of non-conformity to the rules and results of the previous paragraph are those in which this assumption does not hold.

3. The forecasting method.

The procedure for applying the above results needs little explanation. Entering the table of Fig. 8 with the arguments of central pressure and horizontal distance between the 700 mb. and surface centers, one obtains the expected pressure change during the next twenty-four hour period. Admittedly, there are a number of limitations to application of this study. They may be itemized as follows:

a) The presence of a center on the 700 mb. level is required. This need not be a closed center if there is sufficient evidence, of the nature of an inconsistent shallowness of pressure gradient, to indicate reliably the existence of an incipient center.

b) Every effort should be made to ascertain, from the available reports, the most probable position of the lowest point of the 700 mb. surface, since reliance on the geometric center of the lowest closed isophypse will often introduce error.

c) Considerable difficulty will be encountered in the case of a single upper center associated with two adjoining cyclones on the surface. Unless this occurs in a region in which upper-air soundings are relatively numerous, a correct solution is improbable.

To summarize; extreme care must be taken with the 700 mb. analysis to insure sufficient accuracy for the application of this system.

It is believed that this forecasting tool will find its greatest application to the prediction of pressure tendency of those severe, winter storms which, originating in Western Canada, recurve in the Chicago-New York area.

4. Possible further investigation.

A number of avenues of further study are opened by the results obtained above. It might prove interesting, for example, to inquire into the possibility of a similar relationship existing in the case of anticyclones.

A great deal of further work could be done in connection with paragraph 2 above. Actual statistics on the dimensions of the surface and 700 mb. centers in conjunction with the central surface pressure and tendency would allow at least three correlations to be investigated:

a) The calculated horizontal distance between the two centers with respect to the observed distance.

b) The calculated level of non-divergence with respect to the observed pressure tendency.

c) The discrepancy between this calculated level and 13,800 ft. with respect to the difference between the computed pressure change and that observed.

Such lines of inquiry are, of course, in addition to the verification by trial of these results and their integration into the theory of dynamic meteorology.

Had time permitted the author would have been interested in incorporating the wind and density at stations to the southeast a distance of one-half the radius of the surface center in each of the above cases into the correlations as further representations of the advective term of the tendency equation.

In conclusion, it is felt that the principal relationship obtained by this inquiry, as presented in Fig. 8, is of a sufficient high degree of reliability to permit its use as a quantitative forecasting tool, bearing in mind the limitations set forth in the preceding section.

1. Statistical computations.

All correlation calculations in the paper were based on standard statistical principles as defined by Kenney (8). Basically, this presumes the assumption of linear relationship between two-covariates. Therefore, positive correlation signifies a mutual increase between them, or a decrease of one as the other decreases. The simple correlation coefficient is defined as:

$$r_{X_1X_2} = \frac{\frac{1}{N} \sum (X_1 - \overline{X}_1)(X_2 - \overline{X}_2)}{\sigma_{X_1} \quad \sigma_{X_2}}$$

where X_1 and X_2 are the co-variates. The arithmetic mean of each are represented by $\overline{X}_1$ and $\overline{X}_2$ respectively. The standard deviations of X_1 and X_2 from the mean are σx_1 and σx_2 and are the result of the formula.

$$\sigma_x = \left[\frac{1}{N} \sum (x^2 - \overline{x}^2) \right]^{1/2}$$

where N is the number of co-variates under consideration.

The line representing the linear regression of two variates is expressed in the form.

$$(x_1 - \overline{x}_1) = r_{x_1x_2} \frac{\sigma_{x_1}}{\sigma_{x_2}} (x_2 - \overline{x}_2)$$

"Regression of X_1 on X_2" merely states that an estimate of X_1 may be obtained from a known value of X_2 through the use of the above formula in the form of $X_1 = aX_2 + b$. It should be emphasized that this equation, as so developed, is valid only for the estimation of X_1 from X_2 and does not permit its use to estimate X_2.

The standard error of estimate, S, is defined as the mean of the differences between an observed X_1 and the estimated values of X_1 from above. It is a measure of the mean error involved when such an equation is used to estimate X_1 from X_2 and, hence, may be considered as a standard for judging the reliability of the derived relation. S is, by definition, expressed as

$$S_{x_1 x_2} = \sigma_{x_1} \left(1 - r^2_{x_1 x_2}\right)^{1/2}$$

The calculation of multiple correlations involving three and four variates, as accomplished in this study, is also based on the assumption of linear regression. Such multiple correlation coefficients are expressed by the relation:

$$r_{x_1 \cdot x_2 x_3 \ldots\ldots x_y} = \left(1 - \frac{R}{R_{11}}\right)^{1/2}$$

where R is the determinant:

$$
R = \begin{vmatrix}
r_{11} & r_{12} & r_{13} & -- & -- & r_{1y} \\
r_{21} & r_{22} & r_{23} & -- & -- & r_{2y} \\
r_{31} & r_{32} & r_{33} & -- & -- & r_{2y} \\
-- & -- & -- & -- & -- & -- \\
-- & -- & -- & -- & -- & -- \\
r_{y1} & r_{y2} & r_{y3} & -- & -- & r_{yy}
\end{vmatrix}
$$

and R_{11} is the minor of r_{11}:

$$
R_{11} = \begin{vmatrix}
r_{22} & r_{23} & -- & -- & r_{2y} \\
r_{32} & r_{33} & -- & -- & r_{3y} \\
-- & -- & -- & -- & -- \\
-- & -- & -- & -- & -- \\
r_{y2} & r_{y3} & -- & -- & r_{yy}
\end{vmatrix}
$$

The standard error of estimate of X_1 on the basis of X_2 , X_3 , --, --, X_y is defined as

$$
S_{1\cdot 23\ldots y} = \sigma_1 \left(\frac{R}{R_{11}}\right)^{1/2}
$$

And finally, the regression equation of X_1 on X_2, X_3, --, --, X_y is given by:

$$(X_1 - \bar{X}_1)\frac{R_{11}}{\sigma_{X_1}} + (X_2 - \bar{X}_2)\frac{R_{12}}{\sigma_{X_2}} + (X_3 - \bar{X}_3)\frac{R_{13}}{\sigma_{X_3}} + \text{- - - - - - - -} + (X_y - \bar{X}_y)\frac{R_{1y}}{\sigma_{X_y}} = 0$$

2. "Skill Score" computations, #7.

This is a statistical method for eliminating from a purely percentage computation of the favorable distribution of a particular tetrachoric pattern, that part of such a distribution which might result from chance. In effect, then, it is a percentage measure of the improvement of the given pattern over such a chance arrangement.

The Skill Score, SS, is arrived at as follows:

Given a tetrachoric distribution as:

X_1	X_2
Y_1	Y_2

then

$$SS = \frac{(X_1 + Y_2) - E_c}{\Sigma X + \Sigma Y - E_c}$$

where E_c is the number of cases estimated to have been included in the favorable quadrants by chance, and is defined as:

$$E_c = \frac{(\Sigma X)^2 + (\Sigma Y)^2}{\Sigma X + \Sigma Y}$$

It may easily be verified that, on the basis of the above development, a "pure chance" arrangement, i.e., $X_1 = X_2 = Y_1 = Y_2$, would result in a Skill Score of SS = 0.0%.

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